

On the Shift Value Set of Cyclic Shifted Sequences for PAPR Reduction in OFDM Systems

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Abstract—Orthogonal frequency division multiplexing (OFDM) signals have high peak-to-average power ratio (PAPR), which causes distortion when OFDM signal passes through a nonlinear high power amplifier. A partial transmit sequence (PTS) scheme is one of the typical PAPR reduction methods. A cyclic shifted sequences (CSSs) scheme is evolved from the PTS scheme to improve the PAPR reduction performance, where OFDM signal subsequences are cyclically shifted and combined to generate alternative OFDM signal sequences. The shift value (SV) sets in the CSS scheme should be carefully selected because those are closely related to the PAPR reduction performance of the CSS scheme. In this letter, we propose some criteria to select the good SV sets and verify its validness through simulations.

Index Terms—Cyclic shifted sequences (CSS), orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), partial transmit sequence (PTS).

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) is a multicarrier modulation method utilizing the orthogonality of subcarriers. OFDM has been adopted as a standard modulation method in many wireless communication systems such as digital audio broadcasting (DAB), digital video broadcasting (DVB), IEEE 802.11 wireless local area network (WLAN), and IEEE 802.16 wireless metropolitan area network (WMAN). Similar to other multicarrier schemes, OFDM has a high peak-to-average power ratio (PAPR) problem, which makes its implementation quite costly. Thus, it is highly desirable to reduce the PAPR of OFDM signal sequences. Over the last few decades, various schemes to reduce the PAPR of OFDM signal sequences have been proposed such as clipping, coding, active constellation extension (ACE) [1], tone reservation (TR), partial transmit sequence (PTS) [2], and selected mapping (SLM) [3], [4].

Like the SLM scheme, the PTS scheme statistically improves the characteristic of the PAPR distribution of OFDM signals without signal distortion. In the PTS scheme, the input symbol sequence is partitioned into a number of disjoint input symbol subsequences. Inverse fast Fourier transform (IFFT) is then applied to each input symbol subsequence and the resulting OFDM signal subsequences are combined after being multiplied by a set of rotation factors. Next the PAPR is computed for each resulting sequence and then the OFDM signal sequence with the minimum PAPR is transmitted.

Hill *et al.* proposed a cyclic shifted sequences (CSS) scheme, where cyclic shift is used instead of multiplying a rotation factor to the OFDM signal subsequences [5], [6]. It is known that the CSS scheme is better than the PTS scheme from every aspect. First, its PAPR reduction performance is better than the PTS scheme's [3].

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Even though the authors in [3] analyzed the Class-III SLM scheme, the Class-III SLM scheme can be viewed as a combination of the PTS and CSS schemes, where it uses both cyclic shift and multiplication of rotation factors to the OFDM signal subsequences as in [8]. They showed that the cyclic shift of OFDM signal subsequences generates alternative OFDM signal sequences having lower dependency than the case when we multiply the rotation factors to the OFDM signal subsequences. In our simulation results, the PAPR reduction performance comparison between the CSS scheme and the PTS scheme is included, where we re-confirmed that the CSS scheme shows a better PAPR reduction performance than the PTS scheme's when they have the same computational complexity (the number of subblocks) and use the same number of alternative OFDM signal sequences. Second, it is possible to recover the transmitted OFDM signal sequence without side information using some additional techniques at the receiver only [7], [9]. Unlike the CSS scheme, the PTS scheme can do that only if it uses some additional techniques at both the transmitter and the receiver as in [10] and [11].

In this letter, we investigate how to select the shift value (SV) sets in order to boost the PAPR reduction performance of the CSS scheme. We introduce some criteria to select the good SV sets considering the autocorrelation function (ACF) of OFDM signal subsequences, and then verify its validness through simulations.

II. PRELIMINARIES

A. OFDM System and PAPR

In an OFDM system, an OFDM signal sequence in time domain is generated by IFFT as

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi kn}{N}} \quad (1)$$

where N is the number of subcarriers, $X = \{X(0), X(1), \dots, X(N-1)\}$ is an input symbol sequence in frequency domain, and $x = \{x(0), x(1), \dots, x(N-1)\}$ is an OFDM signal sequence in time domain. The PAPR of the OFDM signal sequence x is defined as

$$\text{PAPR} = \frac{\max_{0 \leq n < N} |x(n)|^2}{E[|x(n)|^2]} \quad (2)$$

where $E[\cdot]$ represents the expectation.

B. Cyclic Shifted Sequences (CSS)

Fig. 1 shows a block diagram of the CSS scheme testing U alternative OFDM signal sequences in total [5]. In the CSS scheme, X is divided by a certain partitioning pattern into V disjoint subblocks, input symbol subsequences X_1, X_2, \dots, X_V . Then IFFT converts the V subblocks in frequency domain to the V OFDM signal subsequences in time domain x_1, x_2, \dots, x_V , where $x_v = \{x_v(0), x_v(1), \dots, x_v(N-1)\}$, $1 \leq v \leq V$. For simplicity, we assume that both N and V are integers of power of two. After that, the V OFDM signal subsequences are cyclically shifted and combined

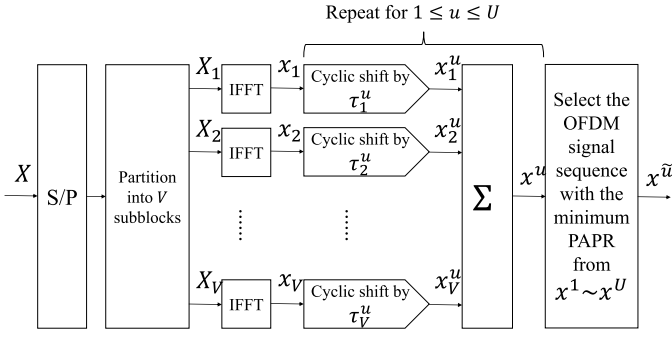


Fig. 1. A block diagram of the CSS scheme [5].

together to make the u -th ($1 \leq u \leq U$) alternative OFDM signal sequence as

$$x^u = \sum_{v=1}^V x_v^u \quad (3)$$

where x_v^u denotes the leftward cyclically shifted version of x_v by some integer τ_v^u ($1 \leq v \leq V$). That is,

$$x_v^u = \{x_v(\tau_v^u), x_v(\tau_v^u + 1), \dots, x_v(N-1), x_v(0), \dots, x_v(\tau_v^u - 1)\}. \quad (4)$$

As the SLM or PTS schemes, the candidate with the lowest PAPR, x^u , is chosen by exhaustive search for transmission with $\lceil \log_2 U \rceil$ bits side information. By using some additional techniques at the receiver, the side information can be recovered [7].

The cyclic shift operation does not destroy the orthogonality between the input symbols $X(k)$'s because, as we all know, cyclic shifting in time domain is equivalent to multiplying a corresponding linear phase vector in frequency domain [5]. In this letter, we denote τ_v^u as a *shift value* and also denote $\bar{\tau}^u = \{\tau_1^u, \tau_2^u, \dots, \tau_V^u\}$ as a *SV set* for the u -th alternative OFDM signal sequence. Clearly, we have to construct U SV sets ($\bar{\tau}^1, \bar{\tau}^2, \dots, \bar{\tau}^U$) to implement the CSS scheme testing U alternative OFDM signal sequences.

Like the PTS scheme, the CSS scheme can use three partition methods, i.e., random, adjacent, and interleaved partition methods. It is widely known that the random partition method gives the best PAPR reduction performance among them while the interleaved partition method gives the worst PAPR reduction performance but it needs the lowest computational complexity. (The interleaved partition method requires approximately one N -point IFFTs to generate U alternative OFDM signal subsequences regardless of U , but the random partition and adjacent partition method needs U N -point IFFTs.) The adjacent partition method is not meaningful practically because it needs a high computational complexity as the random partition needs, but it shows worse PAPR reduction performance than the random partition case. In this letter, we consider the three partition methods.

III. DESIRABLE SHIFT VALUE SETS IN THE CSS SCHEME

In the CSS scheme, the PAPR reduction performance depends on how to construct U SV sets $\{\bar{\tau}^1, \bar{\tau}^2, \dots, \bar{\tau}^U\}$. Considering the fact that the true objective of the CSS scheme is to reduce the probability of the PAPR exceeding some threshold level rather than to reduce the PAPR of each alternative OFDM signal sequence itself, we may say in general that U SV sets that make alternative OFDM signal sequences as statistically independent as possible can perform well.

There are N^V cases of one SV set. That is, $\bar{\tau}^u = \{\tau_1^u, \tau_2^u, \dots, \tau_V^u\}$ can be varied from $\{0, 0, \dots, 0\}$ to $\{N-1, N-1, \dots, N-1\}$. Among these N^V possible SV sets, we select only U SV sets in the CSS

scheme. In general, N^V is a huge number and thus it is hard to design U SV sets without any criterion, which motivates us to propose criteria to select good U SV sets in this letter. By using the deterministic SV sets, which will be made in this letter, we can guarantee the PAPR reduction performance of the CSS scheme without any risk.

A. Desirable Shift Value Sets Without Consideration of Correlation of OFDM Signal Subsequence Components

In fact, the components in an OFDM signal subsequence are not mutually independent, which will be shown in the next subsection. However for now, we assume that the components in the OFDM signal subsequences are mutually independent for simplicity. That is, we have

$$E[x_{v_1}(n_1) \cdot \{x_{v_2}(n_2)\}^*] = \begin{cases} \sigma^2, & v_1 = v_2 \text{ and } n_1 = n_2 \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

where σ^2 is a component power of an OFDM signal subsequence and $\{\cdot\}^*$ denotes the complex conjugate.

Roughly speaking, in both SLM and PTS schemes, in order to boost their PAPR reduction performance, alternative OFDM signal sequences must have low correlation mutually. Therefore, we may use the results in [12] and [13], which investigate the optimal condition of alternative OFDM signal sequences in SLM schemes, although the CSS scheme is evolved from the PTS scheme.

Firstly, as in [12] and [13], we denote the correlation between the n -th component of the i -th alternative OFDM signal sequence and the m -th component of the j -th alternative OFDM signal sequence as

$$\rho_{i,j}(n, m) = E[x^i(n) \cdot \{x^j(m)\}^*]. \quad (6)$$

In [12] and [13], it is shown that the correlation in (6) only depends on the time difference between n and m . That is, (6) can be expressed as

$$\rho_{i,j}(n, m) = E[x^i(n) \cdot \{x^j(n - \delta \bmod N)\}^*] = \rho_{i,j}(\delta) \quad (7)$$

where $0 \leq \delta \leq N-1$.

The authors in [12] consider the simplest case that there are only two alternative OFDM signal sequences, which are x^1 and x^2 ($U=2$). Also, they show that the PAPR reduction performance of the SLM scheme becomes worse as the maximum value of correlation between x^1 and x^2 , i.e., $\max_{0 \leq \delta \leq N-1} \rho_{1,2}(\delta)$ increases. Likewise, in the CSS scheme case, we consider the simplest case that only two alternative OFDM signal sequences x^1 and x^2 exist ($U=2$), generated by two SV sets $\bar{\tau}^1$ and $\bar{\tau}^2$, respectively.

Without loss of generality, x^1 is the original OFDM signal sequence, which is generated by using the all-zero SV set $\bar{\tau}^1 = \{0, 0, \dots, 0\}$. In this case, we have

$$x^1 = \left\{ \sum_{v=1}^V x_v(0), \sum_{v=1}^V x_v(1), \dots, \sum_{v=1}^V x_v(N-1) \right\}. \quad (8)$$

Also, using (4), x^2 by the SV set $\bar{\tau}^2 = \{\tau_1^2, \tau_2^2, \dots, \tau_V^2\}$ is expressed as

$$x^2 = \left\{ \sum_{v=1}^V x_v(\tau_v^2), \sum_{v=1}^V x_v(\tau_v^2 + 1 \bmod N), \dots, \sum_{v=1}^V x_v(\tau_v^2 + N-1 \bmod N) \right\}. \quad (9)$$

Using (5), (7), (8), and (9), $\rho_{1,2}(\delta)$ is given as

$$\begin{aligned}\rho_{1,2}(\delta) &= E\left[x^1(n) \cdot \left\{x^2(n - \delta \bmod N)\right\}^*\right] \text{ using (7)} \\ &= E\left[x^1(0) \cdot \left\{x^2(-\delta \bmod N)\right\}^*\right] \\ &= E\left[\sum_{v=1}^V x_v(0) \cdot \left\{\sum_{v=1}^V x_v(\tau_v^2 - \delta \bmod N)\right\}^*\right] \text{ using (8)\&(9)} \\ &= \sum_{v=1}^V E\left[x_v(0) \cdot \left\{x_v(\tau_v^2 - \delta \bmod N)\right\}^*\right] \text{ using (5)}\end{aligned}\quad (10)$$

where the value of n does not affect $\rho_{1,2}(\delta)$, and thus we use $n = 0$. Using (5), the inner term in the equation (10) becomes

$$E\left[x_v(0) \cdot \left\{x_v(\tau_v^2 - \delta \bmod N)\right\}^*\right] = \begin{cases} \sigma^2, & \tau_v^2 = \delta \\ 0, & \text{otherwise.} \end{cases} \quad (11)$$

For a set $\bar{\tau}^2 = \{\tau_1^2, \tau_2^2, \dots, \tau_V^2\}$, let α_l denote the number of occurrences of l ($l = 0, 1, \dots, N-1$). Clearly, $\alpha_0 + \alpha_1 + \dots + \alpha_{N-1} = V$. Then, using (10) and (11), we have

$$\rho_{1,2}(\delta) = \alpha_\delta \sigma^2. \quad (12)$$

Therefore, the best way to reduce the peak of $\rho_{1,2}(\delta)$ is to satisfy $\alpha_0, \alpha_1, \dots, \alpha_{N-1} \leq 1$, which guarantees $\max_{0 \leq \delta \leq N-1} \rho_{1,2}(\delta) = \sigma^2$. In other words, the relative distances $\tau_v^1 - \tau_v^2$ for all v 's have to be distinct from each other. When $U > 2$, this has to be guaranteed for all possible SV set pairs out of U SV sets. Now we obtain the following criterion.

Criterion 1: Suppose that we have U SV sets; For every (i, j) pair out of the U SV sets ($i \neq j$), the pair should satisfy the condition that the relative distances $\tau_v^i - \tau_v^j \bmod N$ are distinct from each other for all v 's.

Note that the *Criterion 1* is valid when the components in all alternative OFDM signal subsequences are mutually independent. However, actually the OFDM signal subsequence components are not mutually independent because the corresponding input symbol subsequences in frequency domain have $N - N/V$ zeros. That is, it is widely known that the independent input components make independent output components while they pass the IFFT. However, $N - N/V$ zeros means that there are correlations between input components, which causes the dependency between the output components.

B. ACF of OFDM Signal Subsequences

Let S_v be the discrete power spectrum of the v -th OFDM signal subsequence x_v , namely,

$$S_v = \{p(0), p(1), \dots, p(N-1)\} \quad (13)$$

where $p(k) \triangleq E[|X_v(k)|^2]$, and $p(k)$ can have the value of zero or one. This is due to the assumption that the modulation order of all subcarriers is equal and the average power is normalized to one. For example, if the interleaved partition is used, $S_1 = \{10101010\}$ and $S_2 = \{01010101\}$ when $N = 8$ and $V = 2$.

Then the ACF $R_{x_v}(m)$ is given by inverse discrete Fourier transform (IDFT) of S_v . Considering the input symbol sequence X_v has $N - N/V$ zeros in a certain pattern, the corresponding ACF $R_{x_v}(m)$ has a specific shape. Here we investigate only the magnitude of the ACF because the high peak of the OFDM signal sequence is closely related to the magnitude of components.

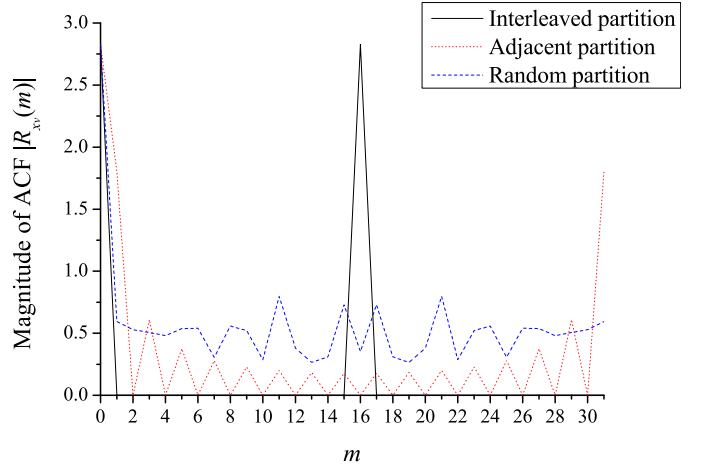


Fig. 2. Magnitude of ACFs for different partition cases.

1) *For Interleaved Partition:* In this case, S_v is an impulse train with an interval of V . Then, the ACF also becomes the impulse train as [14]

$$|R_{x_v}(m)| = \begin{cases} \frac{\sqrt{N}}{V} & \text{if } m = 0 \bmod \frac{N}{V} \\ 0 & \text{otherwise.} \end{cases} \quad (14)$$

2) *For Adjacent Partition:* In this case, S_v is a rectangular function with a width of N/V . Then the ACF becomes the function as [14]

$$|R_{x_v}(m)| = \begin{cases} \frac{\sqrt{N}}{V} & \text{if } m = 0 \\ \frac{\sqrt{N} \sin(m\pi/V)}{\sqrt{N} \sin(m\pi/N)} & \text{if } m \neq 0. \end{cases} \quad (15)$$

3) *For Random Partition:* In this case, S_v can be viewed as a binary pseudo random sequence. Then the ACF has a shape similar to a delta function, where the components except $m = 0$ are close to zero.

Fig. 2 shows an example of the magnitudes of ACFs corresponding to the following power spectrum when $N = 32$ and $V = 2$; $S_1 = \{1010 \dots 1010\}$ for an interleaved partition; $S_1 = \{11 \dots 1100 \dots 00\}$ for an adjacent partition; $S_1 = \{1001011001111000110111010100000\}$ for a random partition, which is a one zero padded m-sequence with length 31; Clearly, S_2 is a complement of S_1 in each partition case, and the shapes of $|R_{x_v}(m)|$ for $v = 1$ and $v = 2$ are same.

C. Desirable Shift Value Sets With Consideration of ACF of OFDM Signal Subsequences

Now we investigate the desirable SV sets with consideration of ACF of the OFDM signal subsequence for three partition cases.

1) *For Random Partition:* In this case, the shape of the ACF is similar to a delta function. Therefore, the *Criterion 1* can be valid criterion.

2) *For Interleaved Partition:* The impulse train ACF in (14) means that components in the OFDM signal subsequence are related to each other as

$$|E[x_{v_1}(n_1) \cdot \{x_{v_2}(n_2)\}^*]| = \begin{cases} \sigma^2, & v_1 = v_2 \text{ and } n_1 = n_2 \bmod \frac{N}{V} \\ 0, & \text{otherwise.} \end{cases} \quad (16)$$

Then, in this case, the magnitude of the inner term in the equation (10) becomes

$$|E[x_v(0) \cdot \{x_v(\tau_v^2 - \delta \bmod N)\}^*]| = \begin{cases} \sigma^2, & \tau_v^2 = \delta \bmod \frac{N}{V} \\ 0, & \text{otherwise.} \end{cases} \quad (17)$$

For a set $\bar{\tau}^2 = \{\tau_1^2, \tau_2^2, \dots, \tau_V^2\}$, let β_l denote the number of occurrences of l after modulo N/V operation for $l = 0, 1, \dots, N/V - 1$. Clearly, $\beta_0 + \beta_1 + \dots + \beta_{N/V-1} = V$. For example, if $\bar{\tau}^2 = \{N/V, N/V, \dots, N/V\}$, then $\beta_0 = V$ and $\beta_1 = \dots = \beta_{N/V-1} = 0$. Consequently, using (10) and (17), we have

$$\rho_{1,2}(\delta) \leq \beta_{\delta \bmod \frac{N}{V}} \sigma^2. \quad (18)$$

The best way to reduce the peak of $\rho_{1,2}(\delta)$ is to satisfy $\beta_0, \beta_1, \dots, \beta_{N/V-1} \leq 1$, which guarantees $\max_{0 \leq \delta \leq N-1} \rho_{1,2}(\delta) = \sigma^2$. Therefore, *Criterion 1* has to be slightly modified as follows.

Criterion 2: Suppose that we have U SV sets; For every (i, j) pair out of the U SV sets ($i \neq j$), the pair should satisfy the condition that the relative distances $\tau_v^i - \tau_v^j \bmod N/V$ are distinct from each other for all v 's.

3) *For Adjacent Partition:* Like the proofs of *Criterion 1* and *Criterion 2*, we may also derive the optimal condition of the U SV sets in this case. However, it may be very complicated work because the inner term in the equation (10) becomes complicated, which is not the simple case with zero or one. Therefore, we give a rough criterion for the adjacent partition case based on the rough interpretation of (15). We think that the adjacent partition is useless in practice, so the rough criterion is enough. In this case, the shape of the ACF in (15) is similar to a sinc function. Then the inner term in the equation (10) becomes smaller as $\tau_v^2 - \delta \bmod N$ gets closer to $N/2$. Therefore, the constraint that the relative distances have to be distinct from each other in *Criterion 1* should be changed into a stronger constraint as follows.

Criterion 3: Suppose that we have U SV sets; For every (i, j) pair out of the U SV sets ($i \neq j$), the pair should satisfy the condition that the relative distances $\tau_v^i - \tau_v^j \bmod N$ are distinct from each other for all v 's; Furthermore, the mutual differences of the V relative distances $(\tau_1^i - \tau_1^j, \tau_2^i - \tau_2^j, \dots, \tau_V^i - \tau_V^j \bmod N)$ should be as close to $N/2$ as possible.

Unfortunately, it is very hard to describe the *Criterion 3* more clearly, but it gives us an important insight to design the good SV sets for the adjacent partition case.

IV. SIMULATION RESULTS

Prior to verifying our proposed criteria for the CSS scheme, the comparison of the PAPR reduction performance between the conventional PTS scheme and the CSS scheme, where we re-confirmed that the CSS scheme shows a better PAPR reduction performance than the PTS scheme's when they have the same computational complexity (the number of subblocks V and a partition method) and use the same number of alternative OFDM signal sequences, $U = 64$. In Fig. 3, $N = 128$, 16-quadrature amplitude modulation (16-QAM), and $V = 4$ are used. The random partition method and the interleaved partition method are used for the both schemes. The PTS scheme uses four rotation factors $\{\pm 1 \pm j\}$. (Note that the number of alternative OFDM signal sequences of the PTS scheme is $U = W^{V-1}$, where W is the number of the rotation factors. The CSS scheme uses the well-designed U SV sets satisfying *Criterion 1* and *Criterion 2*.)

Now, to verify the above proposed criteria for the CSS scheme are valid, we construct the U SV sets in two different ways. That is, the solid lines in Fig. 4 show the PAPR reduction performance of the case when the U SV sets satisfy the above criteria well. On the other hand, the dotted lines in Fig. 4 show the PAPR reduction performance of the case that does not. In the simulations, we use $N = 128$, $U = 4$, and $V = 4$ in common. The 16-QAM is used for all following simulations.

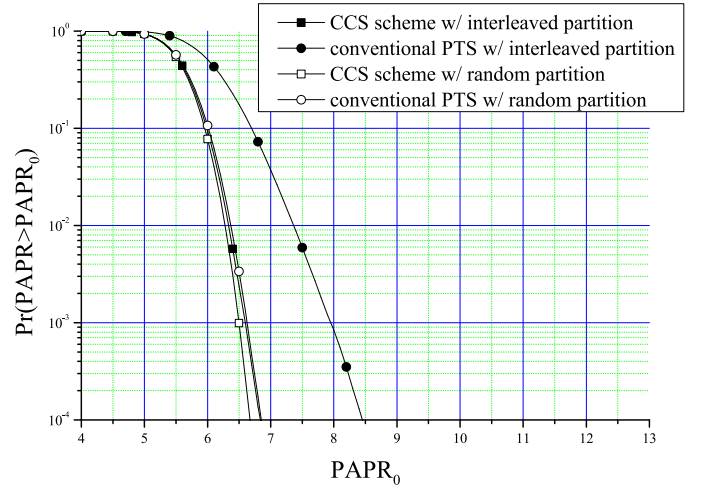


Fig. 3. Comparison of the PAPR reduction performance between the conventional PTS scheme and the CSS scheme when $N = 128$ and $U = 64$.

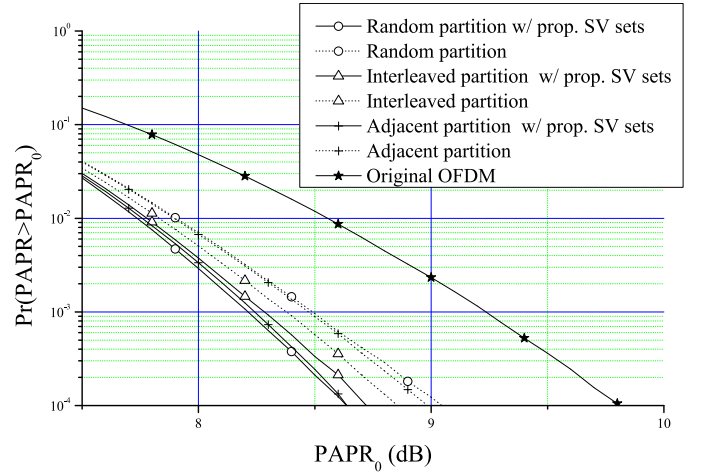


Fig. 4. Comparison of the PAPR reduction performance of the CSS scheme for three partition cases, which are random, interleaved, and adjacent partition cases when $N = 128$, $U = 4$, and $V = 4$ according to the used SV sets.

A. For Random Partition

The SV sets $\bar{\tau}^1 = \{0, 0, 0, 0\}$, $\bar{\tau}^2 = \{0, 8, 16, 24\}$, $\bar{\tau}^3 = \{0, 16, 32, 48\}$, and $\bar{\tau}^4 = \{0, 24, 48, 72\}$ are used for the solid line, which satisfies *Criterion 1*. On the other hand, the SV sets $\bar{\tau}^1 = \{0, 0, 0, 0\}$, $\bar{\tau}^2 = \{0, 4, 8, 12\}$, $\bar{\tau}^3 = \{0, 16, 20, 24\}$, and $\bar{\tau}^4 = \{0, 28, 32, 36\}$ are used for the dotted line, which does not satisfy *Criterion 1*. In Fig. 4, we can verify that the *Criterion 1* for the random partition case is valid.

B. For Interleaved Partition

The SV sets $\bar{\tau}^1 = \{0, 0, 0, 0\}$, $\bar{\tau}^2 = \{0, 1, 2, 3\}$, $\bar{\tau}^3 = \{0, 2, 4, 6\}$, and $\bar{\tau}^4 = \{0, 3, 6, 9\}$ are used for the solid line, which satisfies *Criterion 2*. On the other hand, the SV sets $\bar{\tau}^1 = \{0, 0, 0, 0\}$, $\bar{\tau}^2 = \{0, 8, 16, 24\}$, $\bar{\tau}^3 = \{0, 16, 32, 48\}$, and $\bar{\tau}^4 = \{0, 24, 48, 72\}$ are used for the dotted line, which does not satisfy *Criterion 2* (but still satisfies *Criterion 1*). In Fig. 4, we can verify that the *Criterion 2* for the interleaved partition case is valid.

C. For Adjacent Partition

The SV sets $\bar{\tau}^1 = \{0, 0, 0, 0\}$, $\bar{\tau}^2 = \{0, 44, 73, 95\}$, $\bar{\tau}^3 = \{0, 9, 35, 84\}$, and $\bar{\tau}^4 = \{0, 25, 45, 110\}$ are used for the solid

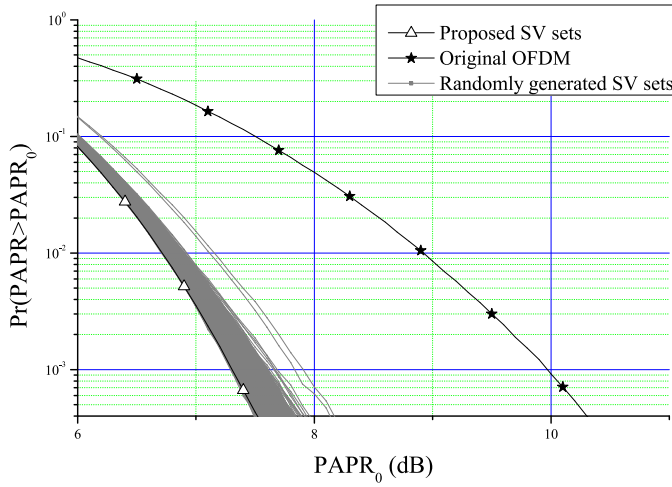


Fig. 5. The optimality of the proposed SV sets when $N = 32$, $U = 4$, interleaved partition, and $V = 4$ are used.

line, which satisfies *Criterion 3* well. On the other hand, the SV sets $\bar{\tau}^1 = \{0, 0, 0, 0\}$, $\bar{\tau}^2 = \{0, 1, 2, 3\}$, $\bar{\tau}^3 = \{0, 2, 4, 6\}$, and $\bar{\tau}^4 = \{0, 3, 6, 9\}$ are used for the dotted line, which does not satisfy *Criterion 3* well (but still satisfies *Criterion 1* and *Criterion 2*). In Fig. 4, we can verify that the *Criterion 3* for the adjacent partition case is valid.

We confirmed that the other SV sets satisfying the proposed criteria except the simulated SV sets above also show the good PAPR reduction performance which is almost same to the solid lines in Fig. 4. In other words, the PAPR reduction performance of the CSS scheme does not depend on the choice of SV sets if they satisfy the proposed criteria.

D. Optimality of the Proposed Criteria

It is hard to compare the PAPR reduction performance of the case using our proposed SV sets to the cases using ALL possible SV sets through simulations because there are too many possible SV sets. Instead, in Fig. 5, we drew 1024 curves using randomly generated 1024 SV sets, where $N = 32$, $U = 4$, interleaved partition, and $V = 4$ are used. Also, we drew the curve using proposed SV sets satisfying *Criterion 2*. The SV sets $\bar{\tau}^1 = \{0, 0, 0, 0\}$, $\bar{\tau}^2 = \{0, 1, 2, 3\}$, $\bar{\tau}^3 = \{0, 2, 4, 6\}$, and $\bar{\tau}^4 = \{0, 3, 6, 9\}$ are used for the proposed SV sets. 100,000 randomly generated input symbol sequences are simulated to draw each curve. In Fig. 5, we can observe the best PAPR reduction performance of the proposed SV sets.

E. Systematic SV Sets Generation

The U SV sets satisfying the proposed criteria can be determined in advance. Thus, exhaustive generation and refinement based on the criteria can be one of the methods to generate good SV sets.

Besides, there may be several deterministic methods to generate the good SV sets. We introduce one of the methods to generate good SV sets satisfying *Criterion 1*. If we set $\tau_v^u = (u-1)(v-1)$, then for practical values of N , V , and U , *Criterion 1* can be satisfied. Let us consider the v_1 -th and v_2 -th subblocks in *Criterion 1* ($v_1 \neq v_2$). Then the relative distances $\tau_{v_1}^i - \tau_{v_2}^j$ mentioned in *Criterion 1* becomes

$$\begin{aligned} (\tau_{v_1}^i - \tau_{v_2}^j) - (\tau_{v_1}^i - \tau_{v_2}^j) &= (v_1 - 1)(i - 1) - (v_1 - 1)(j - 1) \\ &\quad - ((v_2 - 1)(i - 1) - (v_2 - 1)(j - 1)) \\ &= (i - j)(v_1 - v_2). \end{aligned} \quad (19)$$

Since we only consider the case when $1 \leq v_1 \neq v_2 \leq V$ and $1 \leq i \neq j \leq U$, we obtain

$$0 < |(i - j)(v_1 - v_2)| \leq (U - 1)(V - 1). \quad (20)$$

From (19) and (20), the above method is guaranteed to satisfy *Criterion 1* when $(U - 1)(V - 1) < N$ and thus $|(i - j)(v_1 - v_2)| \bmod N \neq 0$. This inequality can be satisfied for practical values of V and U . Besides, the above method is also applicable to generate good SV sets satisfying *Criterion 2*. That is, if $(U - 1)(V - 1) < N/V$, and thus $|(i - j)(v_1 - v_2)| \bmod N/V \neq 0$, which satisfies *Criterion 2* clearly.

V. CONCLUSION

The CSS scheme is the very popular and promising PAPR reduction scheme, which is evolved from the PTS scheme. In this letter, the criteria to select good SV sets are proposed, which can guarantee the optimal PAPR reduction performance of the CSS scheme. The criteria are proposed by considering the ACF of the OFDM signal subsequence for three different partition cases, random, interleaved, and adjacent partition cases. In the simulation results, the CSS scheme using the SV sets satisfying the proposed criteria shows better PAPR reduction performance than the case when the SV sets are not carefully designed.

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